4.3.1 Derivative-based methods for QRS detection

**Problem:** Develop signal processing techniques to facilitate detection of the QRS complex, given that it is the sharpest wave in an ECG cycle.

**Solution 1:** We noted in Section 1.2.4 that the QRS complex has the largest slope (rate of change of voltage) in a cardiac cycle by virtue of the rapid conduction and depolarization characteristics of the ventricles. As the rate of change is given by the derivative operator, the $\frac{d}{dt}$ operation would be the most logical starting point in an attempt to develop an algorithm to detect the QRS complex.

We saw in Section 3.3.3 that the derivative operator enhances the QRS, although the resulting wave does not bear any resemblance to a typical QRS complex. Observe in Figures 3.24 and 3.25 that the slow P and T waves have been suppressed by the derivative operators, while the output is the highest at the QRS. However, given the noisy nature of the results of the derivative-based operators, it is also evident that significant smoothing will be required before further processing can take place.

Balda et al. [99] proposed a derivative-based algorithm for QRS detection, which was further studied and evaluated by Ahlstrom and Tompkins [100], Friesen et al. [101], and Tompkins [27]. The algorithm progresses as follows. In a manner similar to Equation 3.45, the smoothed three-point first derivative $y_0(n)$ of the given signal $x(n)$ is approximated as

$$y_0(n) = |x(n) - x(n - 2)|.$$  \hspace{1cm} (4.1)

The second derivative is approximated as

$$y_1(n) = |x(n) - 2x(n - 2) + x(n - 4)|.$$  \hspace{1cm} (4.2)

The two results are weighted and combined to obtain

$$y_2(n) = 1.3y_0(n) + 1.1y_1(n).$$  \hspace{1cm} (4.3)

The result $y_2(n)$ is scanned with a threshold of 1.0. Whenever the threshold is crossed, the subsequent eight samples are also tested against the same threshold. If at least six of the eight points pass the threshold test, the segment of eight samples is taken to be a part of a QRS complex. The procedure results in a pulse with its width proportional to that of the QRS complex; however, the method is sensitive to noise.

**Illustration of application:** Figure 4.2 illustrates, in the top-most trace, two cycles of a filtered version of the ECG signal shown in Figure 3.5. The signal was filtered with an eighth-order Butterworth lowpass filter with $f_c = 90\ Hz$, downsampled by a factor of five, and filtered with a notch filter with $f_o = 60\ Hz$. The effective sampling rate is 200\ Hz. The signal was normalized by dividing by its maximum value.

The second and third plots in Figure 4.2 show the derivatives $y_0(n)$ and $y_1(n)$, respectively; the fourth plot illustrates the combined result $y_2(n)$. Observe the relatively high values in the derivative-based results at the QRS locations; the outputs are low or negligible at the P and T wave locations, in spite of the fact that the original signal possesses an unusually sharp and tall T wave. It is also seen that the results
Figure 4.2  From top to bottom: two cycles of a filtered version of the ECG signal shown in Figure 3.5; output $y_0(n)$ of the first-derivative-based operator in Equation 4.1; output $y_1(n)$ of the second-derivative-based operator in Equation 4.2; the combined result $y_2(n)$ from Equation 4.3; and the result $y_3(n)$ of passing $y_2(n)$ through the 8-point MA filter in Equation 3.27.
have multiple peaks over the duration of the QRS wave, due to the fact that the QRS complex includes three major swings: Q - R, R - S, and S - ST base-line in the present example (an additional PQ base-line – Q swing may also be present in other ECG signals).

The last plot in Figure 4.2 shows the smoothed result $y_3(n)$ obtained by passing $y_2(n)$ through the 8-point MA filter in Equation 3.27. We now have a single pulse with amplitude greater than 1.0 over the duration of the corresponding QRS complex. A simple peak-searching algorithm may be used to detect each ECG beat. The net delay introduced by the filters should be subtracted from the detected peak location in order to obtain the corresponding QRS location.

Note that peak searching cannot be performed directly on an ECG signal: the QRS might not always be the highest wave in a cardiac cycle, and artifacts may easily upset the search procedure. Observe also that the ECG signal in the present illustration was filtered to a restricted bandwidth of 90 Hz before the derivatives were computed, and that it is free of base-line drift.

**Solution 2:** Murthy and Rangaraj [102] proposed a QRS detection algorithm based upon a weighted and squared first-derivative operator and an MA filter. In this method, a filtered-derivative operator was defined as

$$g_1(n) = \sum_{i=1}^{N} |x(n - i + 1) - x(n - i)|^2(N - i + 1), \quad (4.4)$$

where $x(n)$ is the ECG signal, and $N$ is the width of a window within which first-order differences are computed, squared, and weighted by the factor $(N - i + 1)$. The weighting factor decreases linearly from the current difference to the difference $N$ samples earlier in time, and provides a smoothing effect. Further smoothing of the result was performed by an MA filter over $M$ points to obtain

$$g(n) = \frac{1}{M} \sum_{j=0}^{M-1} g_1(n - j). \quad (4.5)$$

With a sampling rate of 100 Hz, the filter window widths were set as $M = N = 8$. The algorithm provides a single peak for each QRS complex and suppresses P and T waves.

Searching for the peak in a processed signal such as $g(n)$ may be accomplished by a simple peak-searching algorithm as follows:

1. Scan a portion of the signal $g(n)$ that may be expected to contain a peak and determine the maximum value $g_{\text{max}}$. The maximum of $g(n)$ over its entire available duration may also be taken to be $g_{\text{max}}$.

2. Define a threshold as a fraction of the maximum, for example, $Th = 0.5 \cdot g_{\text{max}}$.

3. For all $g(n) > Th$, select those samples for which the corresponding $g(n)$ values are greater than a certain predefined number $M$ of preceding and suc-
ceeding samples of $g(n)$, that is,

$$\{p\} = [ \ n \ | \ g(n) > Th \ ] \text{ AND }$$

$$[ \ g(n) > g(n - i), i = 1, 2, \ldots, M \ ] \text{ AND }$$

$$[ \ g(n) > g(n + i), i = 1, 2, \ldots, M \ ].$$

(4.6)

The set $\{p\}$ defined as above contains the indices of the peaks in $g(n)$.

Additional conditions may be imposed to reject peaks due to artifacts, such as a minimum interval between two adjacent peaks. A more elaborate peak-searching algorithm will be described in Section 4.3.2.

**Illustration of application:** Figure 4.3 illustrates, in the top-most trace, two cycles of a filtered version of the ECG signal shown in Figure 3.5. The signal was filtered with an eighth-order Butterworth lowpass filter with $f_c = 40 \text{ Hz}$, and down-sampled by a factor of ten. The effective sampling rate is $100 \text{ Hz}$ to match the parameters used by Murthy and Rangaraj [102]. The signal was normalized by dividing by its maximum value.

![Figure 4.3](image)

Figure 4.3 From top to bottom: two cycles of a filtered version of the ECG signal shown in Figure 3.5; output $g_1(n)$ of the weighted and squared first-derivative operator in Equation 4.4; output $g(n)$ of the smoothing filter in Equation 4.5.

The second and third plots in Figure 4.3 show the outputs of the derivative-based operator and the smoothing filter. Observe that the final output contains a single,
smooth peak for each QRS, and that the P and T waves produce no significant output. A simple peak-searching algorithm may be used to detect and segment each beat [102].

### 4.3.2 The Pan-Tompkins algorithm for QRS detection

**Problem:** Propose an algorithm to detect QRS complexes in an ongoing ECG signal.

**Solution:** Pan and Tompkins [103, 27] proposed a real-time QRS detection algorithm based on analysis of the slope, amplitude, and width of QRS complexes. The algorithm includes a series of filters and methods that perform lowpass, highpass, derivative, squaring, integration, adaptive thresholding, and search procedures. Figure 4.4 illustrates the steps of the algorithm in schematic form.

![Block diagram of the Pan-Tompkins algorithm for QRS detection.](image)

**Lowpass filter:** The recursive lowpass filter used in the Pan-Tompkins algorithm has integer coefficients to reduce computational complexity, with the transfer function defined as

\[
H(z) = \frac{1}{32} \left(1 - z^{-6}\right)^2.
\]

(See also Equations 3.37 and 3.38.) The output \(y(n)\) is related to the input \(x(n)\) as

\[
y(n) = 2y(n - 1) - y(n - 2) + \frac{1}{32} [x(n) - 2x(n - 6) + x(n - 12)].
\]

With the sampling rate being 200 Hz, the filter has a rather low cutoff frequency of \(f_c = 11 \, \text{Hz}\), and introduces a delay of 5 samples or 25 ms. The filter provides an attenuation greater than 35 dB at 60 Hz, and effectively suppresses power-line interference, if present.

**Highpass filter:** The highpass filter used in the algorithm is implemented as an allpass filter minus a lowpass filter. The lowpass component has the transfer function

\[
H_{lp}(z) = \frac{1 - z^{-32}}{1 - z^{-1}};
\]

the input – output relationship is

\[
y(n) = y(n - 1) + x(n) - x(n - 32).
\]

The transfer function \(H_{hp}(z)\) of the highpass filter is specified as

\[
H_{hp}(z) = z^{-16} - \frac{1}{32} H_{lp}(z).
\]
Equivalently, the output \( p(n) \) of the highpass filter is given by the difference equation

\[
p(n) = x(n - 16) - \frac{1}{32} [y(n - 1) + x(n) - x(n - 32)],
\]

with \( x(n) \) and \( y(n) \) being related as in Equation 4.10. The highpass filter has a cutoff frequency of 5 Hz and introduces a delay of 80 ms.

**Derivative operator:** The derivative operation used by Pan and Tompkins is specified as

\[
y(n) = \frac{1}{8} [2x(n) + x(n - 1) - x(n - 3) - 2x(n - 4)],
\]

and approximates the ideal \( \frac{d}{dt} \) operator up to 30 Hz. The derivative procedure suppresses the low-frequency components of the P and T waves, and provides a large gain to the high-frequency components arising from the high slopes of the QRS complex. (See Section 3.3.3 for details on the properties of derivative-based filters.)

**Squaring:** The squaring operation makes the result positive and emphasizes large differences resulting from QRS complexes; the small differences arising from P and T waves are suppressed. The high-frequency components in the signal related to the QRS complex are further enhanced.

**Integration:** As observed in the previous subsection, the output of a derivative-based operation will exhibit multiple peaks within the duration of a single QRS complex. The Pan-Tompkins algorithm performs smoothing of the output of the preceding operations through a moving-window integration filter as

\[
y(n) = \frac{1}{N} [x(n - (N - 1)) + x(n - (N - 2)) + \cdots + x(n)].
\]

The choice of the window width \( N \) is to be made with the following considerations: too large a value will result in the outputs due to the QRS and T waves being merged, whereas too small a value could yield several peaks for a single QRS. A window width of \( N = 30 \) was found to be suitable for \( f_s = 200 \) Hz. Figure 4.5 illustrates the effect of the window width on the output of the integrator and its relationship to the QRS width. (See Section 3.3.2 for details on the properties of moving-average and integrating filters.)

**Adaptive thresholding:** The thresholding procedure in the Pan-Tompkins algorithm adapts to changes in the ECG signal by computing running estimates of signal and noise peaks. A peak is said to be detected whenever the final output changes direction within a specified interval. In the following discussion, \( SPKI \) represents the peak level that the algorithm has learned to be that corresponding to QRS peaks, and \( NPKI \) represents the peak level related to non-QRS events (noise, EMG, etc.). **\( THRESHOLD I_1 \)** and **\( THRESHOLD I_2 \)** are two thresholds used to categorize peaks detected as signal (QRS) or noise.

Every new peak detected is categorized as a signal peak or a noise peak. If a peak exceeds **\( THRESHOLD I_1 \)** during the first step of analysis, it is classified as a QRS (signal) peak. If the searchback technique (described in the next paragraph) is used,
Figure 4.5 The relationship of a QRS complex to the moving-window integrator output. Upper plot: Schematic ECG signal. Lower plot: Output of the moving-window integrator. QS: QRS complex width. W: width of the integrator window, given as \( N/f_s \) s. Adapted from Tompkins [27].

The peak should be above \( T H E S H O L D \ I_2 \) to be called a QRS. The peak levels and thresholds are updated after each peak is detected and classified as

\[
SPKI = 0.125 PEAKI + 0.875 SPKI \quad \text{if PEAKI is a signal peak; (4.15)}
\]

\[
NPKI = 0.125 PEAKI + 0.875 NPKI \quad \text{if PEAKI is a noise peak;}
\]

\[
T H E S H O L D \ I_1 = NPKI + 0.25(SPKI - NPKI); \quad (4.16)
\]

\[
T H E S H O L D \ I_2 = 0.5 T H E S H O L D \ I_1.
\]

The updating formula for \( SPKI \) is changed to

\[
SPKI = 0.25 PEAKI + 0.75 SPKI \quad (4.17)
\]

if a QRS is detected in the searchback procedure using \( T H E S H O L D \ I_2 \).

**Searchback procedure:** The Pan-Tompkins algorithm maintains two \( RR \)-interval averages: \( RR \ \text{AVERAGE}_1 \) is the average of the eight most-recent beats, and \( RR \ \text{AVERAGE}_2 \) is the average of the eight most-recent beats having \( RR \) intervals within the range specified by \( RR \ \text{LOW LIMIT} = 0.92 \times RR \ \text{AVERAGE}_2 \) and \( RR \ \text{HIGH LIMIT} = 1.16 \times RR \ \text{AVERAGE}_2 \). Whenever a QRS is not detected for a certain interval specified as \( RR \ \text{MISSED LIMIT} = 1.66 \times RR \ \text{AVERAGE}_2 \), the QRS is taken to be the peak between the established thresholds applied in the searchback procedure.
The algorithm performed with a very low error rate of 0.68%, or 33 beats per hour on a database of about 116,000 beats obtained from 24-hour records of the ECGs of 48 patients (see Tompkins [27] for details).

**Illustration of application:** Figure 4.6 illustrates, in the top-most trace, the same ECG signal as in Figure 4.2. The Pan-Tompkins algorithm as above was implemented in MATLAB. The outputs of the various stages of the algorithm are illustrated in sequence in the same figure. The observations to be made are similar to those in the preceding section on the derivative-based method. The derivative operator suppresses the P and T waves and provides a large output at the QRS locations. The squaring operation preferentially enhances large values, and boosts high-frequency components. The result still possesses multiple peaks for each QRS, and hence needs to be smoothed. The final output of the integrator is a single smooth pulse for each QRS. Observe the shift between the actual QRS location and the pulse output due to the cumulative delays of the various filters. The thresholding and search procedures and their results are not illustrated. More examples of QRS detection will be presented in Sections 4.9 and 4.10.

**Figure 4.6** Results of the Pan-Tompkins algorithm. From top to bottom: two cycles of a filtered version of the ECG signal shown in Figure 3.5 (the same as that in Figure 4.2); output of the bandpass filter (BPF, a combination of lowpass and highpass filters); output of the derivative-based operator; the result of squaring; and 100× the result of the final integrator.